Period 3, Feb 5, 2025

displacement $= \int v(T)\,dT$ distance $= \int |v(T)|\,dT$

4. An object in rectilinear motion is moving along a horizontal line with velocity $v(t) = 3t^2 - 6t$ (in meters per second).

a) Find the total distance the object moved from $t = 1$ to $t = 4$. $|-2| + |20| = 2 + 20 = 22$

b) Find the total displacement of the object from $t = 1$ to $t = 4$. $-2 + 20 = 18 \Rightarrow \int_1^4 (3T^2 - 6T)\,dT = T^3 - 3T^2 \Big|_1^4$

c) Find the average velocity of the object from $t = 1$ to $t = 4$. 6 m/s

d) For what times is the object at rest? Justify your answer. $T=0$ $T=2$

e) At what time (if any) does the particle change directions. Justify your answer.

$1 \to 4$

Average velocity · width = displacement

$\frac{A \cdot \cancel{3}}{\cancel{3}} = \frac{18}{3}$

$A = 6$

$T=0$ $T=2$

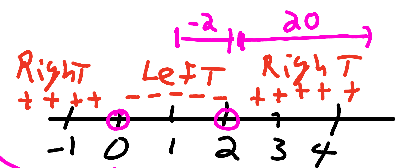
an object is at Rest when $v(T) = 0$

Changes direction when $(v(T))$ changes $(\pm)$ signs. To change signs $v(T) = 0$ or $\nexists$

$V(T) = 3T^2 - 6T = 3T(T-2)$

$V(0) = 3(0)(0-2) = 0$

$V(2) = 3 \cdot 2(2-2) = 6 \cdot 0 = 0$



$V(-1) = 3(-1)^2 - 6(-1) = 3 + 6 = 9$

$V(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$

$V(3) = 3(3)^2 - 6(3) = 27 - 18 = 9$

$\int_1^2 v(T)\,dT = \int_1^2 (3T^2 - 6T)\,dT = T^3 - 3T^2 \Big|_1^2$

Left 2

$2^3 - 3(2)^2 - [1^3 - 3(1)^2]$

$8 - 12 - 1 + 3 = -2$

$\int_2^4 v(T)\,dT = \int_2^4 (3T^2 - 6T)\,dT = T^3 - 3T^2 \Big|_2^4$

$4^3 - 3(4)^2 - [2^3 - 3(2)^2]$

$64 - 48 - 8 + 12 = 76 - 56 = 20$

Right 20

$$\int_3^{3x^2+2x} (5T-2)\,dT = \frac{5}{2}T^2 - 2T \Big[_3^{3x^2+2x}$$

$$\frac{5}{2}(3x^2+2x)^2 - 2(3x^2+2x) - \left[\frac{5}{2}(3)^2 - 2(3)\right]$$

$$\frac{d}{dx}\left[\int_3^{3x^2+2x} (5T-2)\,dT\right] = \left[5(3x^2+2x) - 2\right]\left[6x+2\right]$$

$$\frac{d}{dx}\left[\int_0^{x^3} \sin t\,dt\right] = \frac{d}{dx}\left[-\cos T \Big[_0^{x^3}\right]$$

$$\frac{d}{dx}\left[-\cos x^3 - (-\cos 0)\right]$$

$$\frac{d}{dx}\left[-\cos x^3 + 1\right] = -(-\sin x^3) \cdot 3x^2$$

$$= +(\sin x^3)(3x^2)$$

$$= 3x^2 \sin x^3$$

Example 7:

$$\frac{d}{dx}\left[\int_{x^2}^{3x} f(t)\,dt\right] = [F(3x)] \cdot 3 - [F(x^2)] \cdot 2x$$

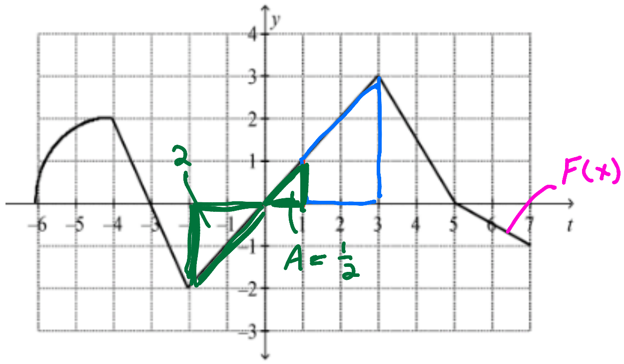
8)

$$\frac{d}{dx}\left[\int_{2x}^{x^2} \frac{1}{2+e^t}\,dt\right] = \frac{2x}{2+e^{x^2}} - \frac{2}{2+e^{2x}}$$

9) $g(x) = \int_{6x}^{4x^2} \sqrt{1+t^4}\,dt$, What is $g'(x)$

$$g'(x) = \sqrt{1+(4x^2)^4} \cdot 8x - \sqrt{1+(6x)^4} \cdot 6$$

Let $g(x) = \int_1^x f(t)\,dt$, where f is the continuous function defined $[-6, 7]$ whose graph is shown below. Find the indicated values.



$g(x) = \int_1^x F(T)dT$

10) $g(1)$

$\int_1^1 F(T)dT = 0$

11) $g(3) = \int_1^3 F(T)\,dT$

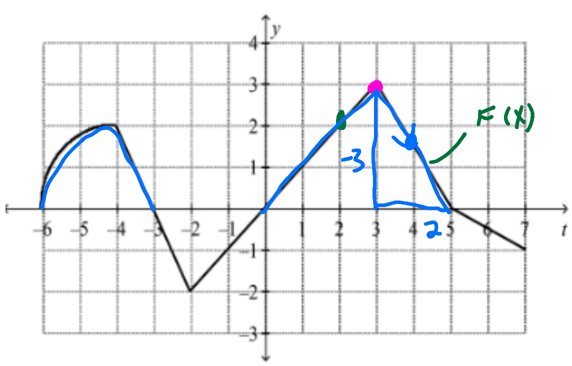
Area under curve

$\frac{1}{2}(1+3)\cdot 2 = 4$

12) $g(-2) = \int_1^{-2} F(T)\,dT$

$-\frac{1}{2} + 2 = \frac{3}{2}$

Let $g(x) = \int_1^x f(t)\,dt$, where f is the continuous function defined $[-6, 7]$ whose graph is shown below. Find the indicated values.



13) $g'(2) =$

$g'(2) = F(2)$

$g'(2) = 2$

$g(x) = \int_1^x F(T)dT$

$g'(x) = F(x)$

$g''(x) = F'(x) =$ Slope of $F(x)$

14) $g'(3) = F(3) = 3$

15) $g''(4) =$ Slope of $F(x)$ at $x=4$

$g''(4) = -\frac{3}{2}$

Let $g(x) = \int_1^x f(t)\,dt$, where f is the continuous function defined $[-6, 7]$ whose graph is shown below. Find the indicated values.

$g'(x) = + = F(x)$ $\quad (-6,-3) \cup (0,5)$

16) Find the intervals where g is increasing. Justify your answer.

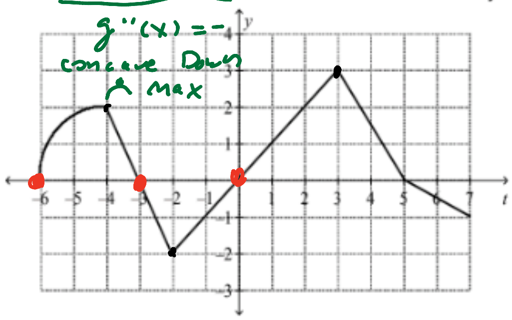
$g'(x) = 0 = F(x)$ $\quad x = -6, -3, 0, 5$

17) Find the x-coordinate of each point where g has a horizontal tangent. For each of these points, determine whether g has a relative minimum, relative maximum or neither. Justify your answer.

$g''(x) = +$

concave up

(Min)



$g''(x) = F'(x) =$ Slope of $F(x)$

$g''(-6) = +$ (Min)

$g''(-3) = - = \frac{-2}{1}$ (Max)

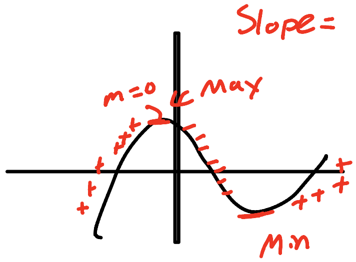
$g''(0) = + = 1$ (Min)

$g''(5) = \phi = -$ in (Max)

$g'(x) = F(x)$

Slope of $g(x) = F(x)$

Slope =



Example 19: The population of the United States is growing at the rate of $P'(t) = 2.867(1.009)^t$ million people per year, where t is the number of years since 2015. If the population of the United States was 320 million people in 2015, what is the projected population in 2020.

Write an expression for the population of the US t years after 2015.

Population 2020

$$\int_0^5 2.867(1.009)^x dx = 14.6609437935$$

$$\text{Population} = 320 + \int_0^5 2.867(1.009)^T dT = 320 + 14.6609$$

$$334.6609$$

Example 21: If the velocity particle is given by $v(t) = 2t$, meters/second and its initial position is 5 meters from the origin, write an expression for the position of the particle at any time t.

$$5 + \int_0^T 2T\,dT = 5 + T^2\Big[_0^T = 5 + T^2 - 0^2 = 5 + T^2$$

Student Example: The rate, in bees per minute, at which bees leave the hive is given by $h(t) = t$ and the rate at which they enter is given by $b(t) = t^2$. If at $t = 3$ min, there are 10 bees in the hive, write an expression for the amount of bees at any given moment of time t. Use your expression to calculate the number of bees at $t = 6$.

Leave $h(T) = T$

Enter $b(T) = T^2$

$$\#\text{of Bees} = 10 - \int_3^T T\,dT + \int_3^T T^2\,dT$$

$$10 = B_0 - \int_0^3 T\,dT + \int_0^3 T^2\,dT \Rightarrow 10 = B_0 - \tfrac{1}{2}T^2\Big[_0^3 + \tfrac{1}{3}T^3\Big[_0^3$$

$$10 = B_0 - \left[\tfrac{1}{2}(3)^2 - \tfrac{1}{2}(0)^2\right] + \left[\tfrac{1}{3}(3)^3 - \tfrac{1}{3}(0)^3\right] \Rightarrow 10 = B_0 - \tfrac{9}{2} + 9 \Rightarrow 1 = B_0 - \tfrac{9}{2} + \tfrac{9}{2}$$

$$-9 \qquad -9 \qquad +\tfrac{9}{2}$$

$$B_0 = 11/2 = 5\tfrac{1}{2}$$

$$\# \text{ of Bees} = 10 - \int_3^T T\,dT + \int_3^T T^2\,dt = 5\tfrac{1}{2} - \int_0^T T\,dT + \int_0^T T^2\,dt$$

$$10 - \left[\tfrac{1}{2}(T^2) - \tfrac{1}{2}(3)^2\right] + \left[\tfrac{1}{3}T^3 - \tfrac{1}{3}(3)^3\right] = 5\tfrac{1}{2} - \tfrac{1}{2}T^2 + \tfrac{1}{3}T^3$$

Example 5

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

Solution

Temperature $= 95 - \int_0^5 6e^{-0.1t}\,dt = 71.392$°C

$T = 0 \quad \text{Temp} = 95$

$$95 - \int_0^5 6e^{-0.1T}\,dT$$

$$95 - 23.608 = 71.392$$

$$\int_0^5 6e^{-.1t}\,dt$$

$= 23.6081604172$